Department of Mathematics Math 108
Tuskegee University Final Examination 2 May 2009

Name:_____ Instructor:_____

CALCULATORS MAY NOT BE SHARED. No items should be on your desk except pens, pencils, calculators, examination and the scratch paper provided by the instructor. You MAY NOT supply your own scratch paper. Keep your work covered at all times. Do not communicate with or borrow items from other students. Keep your eyes on your own paper.

Part I. Multiple Choice. Worth two points each. Blacken the circle corresponding to your answer.

	1	2	3	4	5	6
a	O	O	O	O	O	O
b	O	O	O	O	O	O
c	O	O	O	O	O	O
d	O	O	O	O	O	O

1. The graph of $f(x) = \dfrac{2x-6}{x-1}$ has a vertical asymptote with equation
   (a) $y = 2$
   (b) $x = 1$
   (c) $x = 3$
   (d) $y = 1$

2. The domain of $f(x) = \dfrac{2x-6}{x-1}$ is
   (a) $(1, \infty)$
   (b) $(-\infty, 1) \cap (1, \infty)$
   (c) $(-\infty, 3) \cup (3, \infty)$
   (d) $(-\infty, 1) \cup (1, \infty)$

3. When converted to radian measure $30° =$
   (a) $\frac{\pi}{2}$
   (b) $\frac{\pi}{3}$
   (c) $\frac{\pi}{4}$
   (d) $\frac{\pi}{6}$

4. In a right triangle $\triangle ABC$ if angle $\alpha = 30°$ and side $a = 4$, then the length of the hypotenuse is
   (a) 8
   (b) 4
   (c) 2
   (d) $\frac{8\sqrt{3}}{3}$

5. Which trigonometric functions of θ are positive if θ is in quadrant III?
   (a) $\sin\theta, \csc\theta$
   (b) $\tan\theta, \cot\theta$
   (c) $\cos\theta, \sec\theta$
   (d) $\sin\theta, \cos\theta$

6. Complete the Pythagorean identity: $\sec^2\theta =$
   (a) $\tan^2\theta - 1$
   (b) $1 - \csc^2\theta$
   (c) $1 + \tan^2\theta$
   (d) $1 + \cos^2\theta$

	7	8	9	10	11	12	13	14
a	O	O	O	O	O	O	O	O
b	O	O	O	O	O	O	O	O
c	O	O	O	O	O	O	O	O
d	O	O	O	O	O	O	O	O

7. The reference angle of $150°$ is

   (a) $30°$
   (b) $60°$
   (c) $45°$
   (d) $180°$

8. $\cos(\alpha+\beta) =$

   (a) $\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$
   (b) $\cos(\alpha)\sin(\beta)-\sin(\alpha)\cos(\beta)$
   (c) $\cos(\alpha)\sin(\beta)+\sin(\alpha)\cos(\beta)$
   (d) $\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$

9. Which of the following is NOT equal to $\cos(2\theta)$?

   (a) $\cos^2(\theta)-\sin^2(\theta)$
   (b) $2\cos^2(\theta)-1$
   (c) $2\cos(\theta)$
   (d) $1-2\sin^2(\theta)$

10. In $\triangle ABC$, $\alpha = 38°$, $a = 10$ and $b = 7$. This problem is of type

    (a) SAS
    (b) SSA
    (c) ASA
    (d) SAA

11. Complete the identity: $\sin\left(\frac{x}{2}\right) =$

    (a) $\pm\sqrt{\frac{1-\cos x}{1+\cos x}}$
    (b) $\pm\sqrt{\frac{1-\cos x}{2}}$
    (c) $\frac{1}{2}\sin x$
    (d) $\pm\sqrt{\frac{1+\cos x}{2}}$

12. How many solutions does $2\sin x - 1 = 0$ have in the interval $[0°, 360°)$?

    (a) 0
    (b) 1
    (c) 2
    (d) 4

13. The magnitude of the vector $\langle -5, 12\rangle$ is

    (a) 17
    (b) $\sqrt{119}$
    (c) 7
    (d) 13

14. $\left(3e^{10°i}\right)^2 =$

    (a) $9e^{20°i}$
    (b) $9e^{100°i}$
    (c) $6e^{20°i}$
    (d) $6e^{100°i}$

	15	16	17	18	19	20
a	O	O	O	O	O	O
b	O	O	O	O	O	O
c	O	O	O	O	O	O
d	O	O	O	O	O	O

15. The focus of the parabola $x^2 = -20y$ has coordinates

    (a) $(0, -20)$
    (b) $(0, 20)$
    (c) $(5, 0)$
    (d) $(0, -5)$

16. The ellipse $\frac{x^2}{20} + \frac{y^2}{16} = 1$ has foci

    (a) $(\pm 2, 0)$
    (b) $(0, \pm 2)$
    (c) $(\pm\sqrt{6}, 0)$
    (d) $(0, \pm\sqrt{6})$

17. A hyperbola has transverse axis on the x-axis and the transverse axis has length 6 and the conjugate axis has length 8. Which of the following is the equation of the hyperbola?

    (a) $\frac{x^2}{3} - \frac{y^2}{4} = 1$
    (b) $\frac{x^2}{6} - \frac{y^2}{8} = 1$
    (c) $\frac{x^2}{9} - \frac{y^2}{16} = 1$
    (d) $\frac{x^2}{36} - \frac{y^2}{64} = 1$

18. $\sum_{k=1}^{3}(k^2 - k) =$

    (a) 2
    (b) 6
    (c) 4
    (d) 8

19. An arithmetic sequence has first term $a_1 = 3$ and common difference $d = 2$. The 301st term of the sequence is

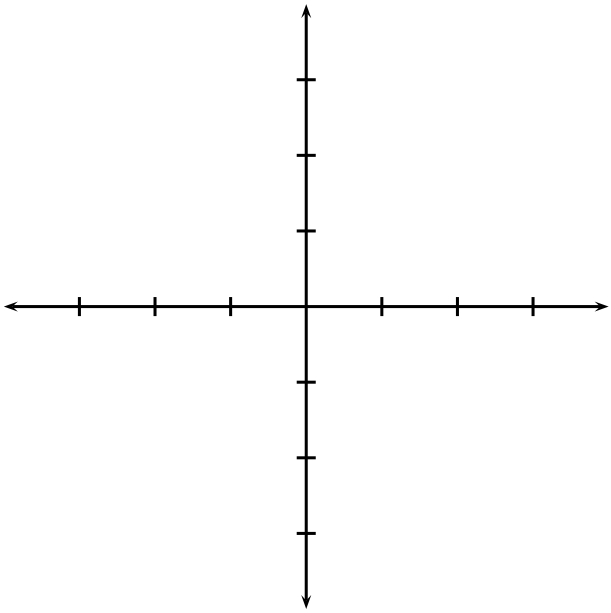
    (a) 45,752
    (b) 603
    (c) 605
    (d) 903

20. The common ratio r of the geometric sequence $12, 6, 3, \cdots$ is

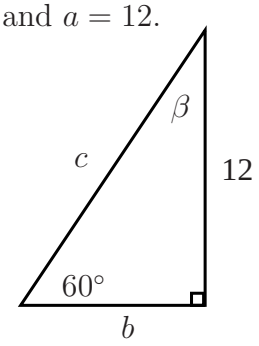
    (a) 12
    (b) 6
    (c) $\frac{1}{2}$
    (d) $\frac{1}{3}$

Part II. Worth 6 points each. Show your work on each of the following.

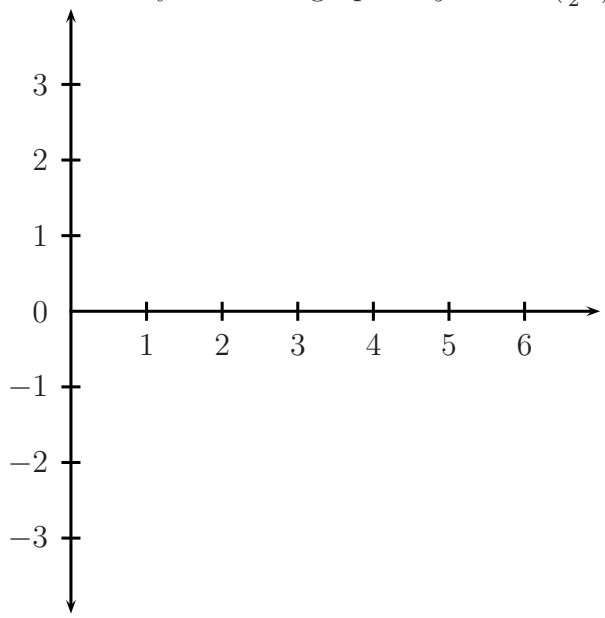
1. Draw a rough sketch of the graph of $y = \dfrac{2x}{x-1}$. Sketch all asymptotes as dotted lines.



2. Given the right triangle $\triangle ABC$ find β, b and the hypotenuse c exactly given $\alpha = 60°$ and $a = 12$.



3. Sketch one cycle of the graph of $y = 2\sin(\frac{\pi}{2}x)$.



4. Verify the identity $\sec x - \cos x = \sin^2 x \sec x$

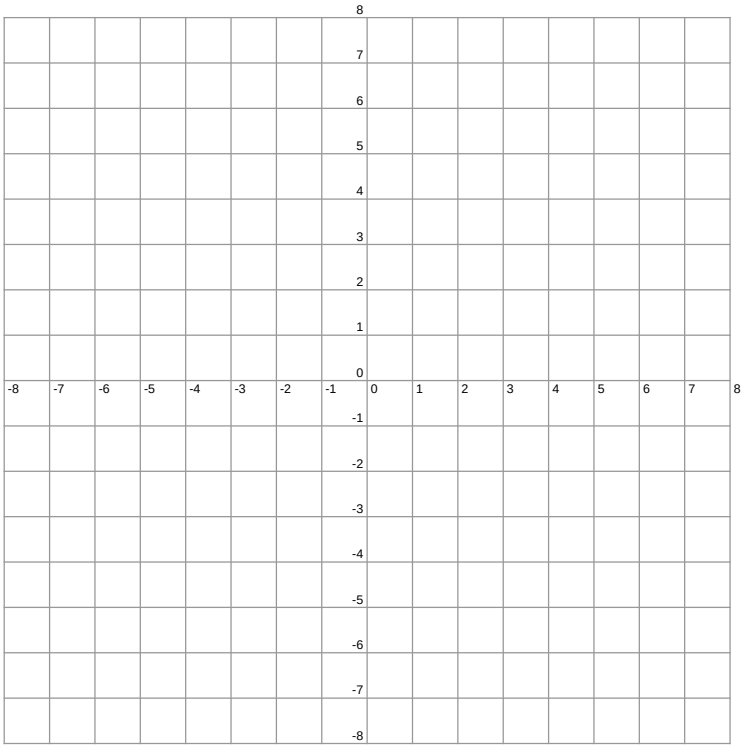
5. Find all solutions in the interval $[0°, 360°)$ for $2\sin\theta + 1 = 0$.

6. Find all solutions of $\triangle ABC$. If there are two solutions, find both solutions.
$\alpha = 32.7°$, $a = 11.2$, $c = 8.7$.

7. Given vectors $\mathbf{u}$ and $\mathbf{v}$ are separated by an angle $\theta = 120^\circ$ and that $|\mathbf{u}| = 7$ and $|\mathbf{v}| = 15$ find the magnitude $|\mathbf{u} + \mathbf{v}|$ of the resultant vector and the angle α between $\mathbf{u}$ and the resultant vector.

8. Convert the complex number $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ to polar form. Use the polar form of $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ to find $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3$. Convert the answer back into rectangular form.

9. Sketch the graph of the hyperbola. Graph the asymptotes and the foci. $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1.$



10. Evaluate the sum of the arithmetic series $\displaystyle\sum_{k=1}^{100}(3k+2) = 5 + 8 + 11 + \cdots + 302 = ?$

Work one of the following bonus problems for an extra 10 points.

1. Find the partial fraction decomposition of $\dfrac{8x-23}{x^2-5x+4}$
2. Find the exact value of $\cos^2\left(\arcsin\left(\frac{5}{7}\right)\right)$
3. Solve $\triangle ABC$ for the angle α given $a = 8.1$, $b = 2.7$ and $\gamma = 12.8°$.
4. Find all complex cube roots of the complex number $-27i$.
5. Find the sum of $S = 2 + 3.5 + 5 + 6.5 + 8 + \cdots + 168.5 + 170$.